Identification of structural design curve using experimental thermal fatigue data

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Abstract: The present paper aims at introducing a complete probabilistic scheme to compute the probability density function of the number-of-cycles to failure of structures at given stress level. The proposed framework combines a specimen random fatigue model, built using fatigue laboratory data, and monitoring data related to pure thermal fatigue specimens that properly represents real components. The proposed method allows to identify the passage sample-to-structure factor using an efficient maximum likelihood approach. The approach is applied to the INTHERPOL testing facility, developed at EDF R&D. These thermal fatigue tests have been carried out on tubular austenitic steel specimen under a specific thermal load. The identified probabilistic fatigue curves related to both specimens and structures are finally compared to one another and to the experimental data.

1 INTRODUCTION

The behavior of structures (e.g. nuclear power plant components) submitted to thermal fatigue is affected by numerous uncertainties, e.g. the random nature of material parameters, the statistical variability of the loading, the scatter observed in fatigue test data. Thus, it is interesting to introduce a global probabilistic framework [1] as an alternative to current nuclear design codes, in order to assess the fatigue lifetime of a component. This requires to identify the probability density functions of all the random variables involved in the thermal fatigue model, especially the structural random number-of-cycles-to-failure $N^{stru}(S,\omega)$ at a given stress level S.

In order to identify the structural random number-of-cycles-to-failure $N^{stru}(S,\omega)$, the first challenge deals with the estimation of the specimen random number-of-cycles-to-failure $N^{spec}(S,\omega)$, using a statistical treatment of fatigue laboratory data. Several studies have been carried out to fit fatigue laboratory data sets by maximum likelihood estimation methods under different assumptions [1,2]. On the other hand, fatigue life tests are performed in idealized conditions (e.g ambient temperature, standard specimens) in laboratory. In the existing nuclear design code, two specimens-to-structure passage factors, γ^N and γ^S, are introduced to relate the number-of-cycles-to-failure of actual reactor components starting from those of laboratory test specimens. The factors account for the effects of size, surface finish, and material and geometric singularities such as welds. The literature shows that these factors are not well known, leading to the introduction of random variables.

This paper aims at introducing:

- a methodology to carry out a statistical treatment of fatigue laboratory data in order to extract a *random specimen fatigue design curve*, starting from a database consisting of 153 austenitic stainless steel specimens ;

- a probabilistic approach that allows to identify specimens to structure passage factor γ^S in the high cycle fatigue life domain. The proposed approach is applied to the INTHERPOL testing facility, developed at EDF R&D [4].

2 STATISTICAL TREATMENT OF FATIGUE LABORATORY DATA

2.1 Principles of the probabilistic characterization of specimen fatigue life

Let $N^{spec}(S,\omega)$ denote the random variable "number-of-cycles-to-failure" observed at stress level S, for a laboratory specimen. The notation ω recalls the randomness of this quantity. At each stress level, $N^{spec}(S,\omega)$ is supposed to be a lognormally distributed random variable. This can be written:

$$\ln N^{spec}(S,\omega) = \lambda(S) + \sigma(S,\omega) \tag{1}$$

where $\lambda(S)$ is the mean value of the logarithm of N^{spec} at stress level S and $\sigma(S,\omega)$ is zero-mean Gaussian random variable. We also suppose that these variables (representing the scatter of ln N^{spec} around its mean-value) are perfectly correlated: this means that a given sample (which is a "realization" of the material in the probabilistic vocabulary) is good or bad with respect to fatigue life whatever the stress level applied. Under this assumption, Eq.(1) may be simplified into:

$$\ln N^{spec}(S,\omega) = \lambda(S) + \sigma(S)u(\omega) \tag{2}$$

where $u(\omega)$ is a standard normal random variable. The mean value $\lambda(S)$ and standard deviation $\sigma(S)$ of ln N^{spec} have to be given a particular form as functions of S.

In this paper, the form of the curve used for the mean $\lambda(S)$ is inspired by the Langer model [5]:

$$\lambda(S) = B + A\ln(S-E) \tag{3}$$

where E is the endurance limit, and A and B are unknown parameters that allow the best fit for the median curve.

In order to take into account the scattering of the endurance limit, parameter E in Eq.(3) is considered itself as a lognormal random variable, with parameters m_V and σ_V, i.e. $E = e^V$, where V is a Gaussian random variable with probability density function (PDF) f_V :

$$f_V(v) = \frac{1}{\sigma_V}\varphi\left(\frac{v-m_V}{\sigma_V}\right) \tag{4}$$

where m_V and σ_V are the mean and the standard deviation of V, and $\varphi(x) = 1/\sqrt{2}\pi\exp\left(-x^2/2\right)$ is the standard normal PDF.

Suppose now that $X = \ln S$ et $Y = \ln N^{spec}$. Conditionally to $V = v$, the random variable Y is gaussian and its PDF $f_{Y|V}$ reads:

$$f_{Y|V}(y,x,A,B,v) = \frac{1}{\sigma(x,A,B,v,\delta)}\varphi\left(\frac{y-\lambda_x(x,A,B,v)}{\sigma(x,A,B,v)}\right) \tag{5}$$

In this equation, $\lambda(x,A,B,v)$ is the mean fatigue curve defined in terms of $S = e^x$ and $E = e^v$ in Eq.(3):

$$\lambda(x,A,B,v) = B + A\ln\left(\exp(x) - \exp(v)\right) \tag{6}$$

Similary, $\sigma(x,A,B,v)$ is the standard deviation of ln N^{spec}. It is supposed to be proportional to the mean value $\lambda(x,A,B,v)$:

$$\sigma(x,A,B,v,\delta) = \delta\ \lambda(x,A,B,v) \tag{7}$$

The marginal density of Y is then given by:

$$f_Y(y,x,A,B,v) = \int_{\Re} f_{Y|V}(y,x,A,B,v)\, f_V(v) \tag{8}$$

which finally reads due to Eqs.(4),(5):

$$f_Y(x,y,A,B,m_V,\sigma_V,\delta) = \int_{-\infty}^{x} \frac{1}{\sigma(x,A,B,v,\delta)}\frac{1}{\sigma_V}\varphi\left(\frac{y-\lambda(x,A,B,v)}{\sigma(x,A,B,v)}\right)\varphi\left(\frac{v-m_V}{\sigma_V}\right)dv \tag{9}$$

In Eq.(9), the upper bound of the integral takes into account the fact that experimental points have sense for $X > V$, i.e. $S > E$.

The marginal cumulative distribution function (CDF) of Y is given by:

$$\begin{aligned} F_Y(x,y,A,B,m_V,\sigma_V,\delta) &= \int_{-\infty}^{y} f_Y(x,t,A,B,m_V,\sigma_V,\delta)\ dt \\ &= \int_{-\infty}^{x} \frac{1}{\sigma_V}\Phi\left(\frac{y-\lambda_x(x,A,B,v)}{\sigma(x,A,B,v)}\right)\varphi\left(\frac{v-m_V}{\sigma_V}\right)dv \end{aligned} \tag{10}$$

where Φ is the standard normal CDF:

$$\Phi(x) = \int_{-\infty}^{x} \varphi(t)\, dt \tag{11}$$

The postulated model requires determining five unknown parameters: m_V and σ_V (distribution of the endurance limit), A and B (form of the median curve), and δ (scatter of the data). These parameters are determined simultaneously using the method of Maximum Likelihood (ML) described below.

2.2 Maximum likelihood estimation

Assuming that experimental points $\left\{y_q = \ln(N_q^{spec}); q = 1,\ldots,Q\right\}$ have been collected for stress levels $\left\{x_q = \ln(S_q); q = 1,\ldots,Q\right\}$, the likelihood of the observations reads:

$$L_Q(A,B,m_V,\sigma_V,\delta) = \prod_{q=1}^{Q} \left\{ \left[f_Y(y_q,x_q,A,B,m_V,\sigma_V,\delta) \right]^{\alpha_q} \left[1 - F_Y(y_q,x_q,A,B,m_V,\sigma_V,\delta) \right]^{1-\alpha_q} \right\} \tag{12}$$

where:

$$\alpha_q = \begin{cases} 1 \text{ if } N_q^{spec} \text{ is a failure} \\ 0 \text{ if } N_q^{spec} \text{ is a censored observation} \end{cases}$$

Moreover f_Y and F_Y are given by Eq.(9) and Eq.(10), respectively.

The determination of parameters m_V, σ_V, a, b and δ is performed by requiring that they maximize the likelihood defined in Eq.(12), or equivalently minimize the following quantity:

$$-2\ln(L_Q) = -2\sum_{q=1}^{Q} \left\{ \alpha_q \ln\left[f_Y(y_q,x_q,A,B,m_V,\sigma_V,\delta) \right] + (1-\alpha_q)\ln\left[1 - F_Y(y_q,x_q,A,B,m_V,\sigma_V,\delta) \right] \right\} \tag{13}$$

The minimization problem described above is solved using classical routines implemented in MathCad [6].

2.3 Application to austenitic stainless steel fatigue data

In this section, the model described in the previous section has been fitted to fatigue data provided by EDF R&D. The data comprises 153 specimens of austenitic stainless steel. Experimentally speaking, fatigue life N is defined as the number of applied cycles at which the stress amplitude in tension side decreases to 75% of the maximum stress amplitude. The dataset also includes 8 censored observations. Figure 1 shows the data and random fatigue models on a log-log scale with number of cycles in the horizontal axis, as is traditional in the plot of *S-N* curves. In this figure, "**O**" and "☐" represent failure and censored observations, respectively.

Figure 1 also shows the estimates of the 0.025, 0.50 and 0.975 quantiles of fatigue life obtained from the model: data seems to be well represented by the model since all data points except 2 are included in the interval. Table I gives the ML estimates of the model parameters and the approximated confidence intervals for each parameter, based on large-sample asymptotics.

Remark: it is to be noted that no experimental values appear on the two obtained curves since the database is private. Only the methodology of statistical treatment is of importance.

Table I. Numerical results of the statistic treatment of austenitic stainless steel data

	A	B	m_V	σ_V	δ
Optimum value	-2.17	22.85	5.55	0.11	0.046
95% confidence interval	(-2,233 ; -2,097)	(22,306 ; 23,384)	(5,445 ; 5,647)	(0,057 ; 0,163)	(0,040 ; 0,052)

Realizations of the number-of-cycles to failure can now be simulated from the specimen random fatigue model $\ln N^{spec}(S,\omega)$, which depends on 3 deterministic parameters (a,b and δ) and 2 random variables (the endurance limit E and a standard normal random variable u).

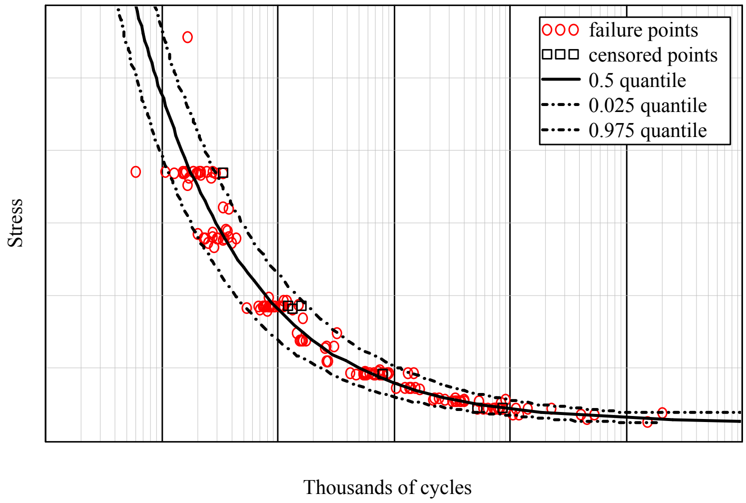

Figure 1. Probabilistic *S-N* (0.025, 0.50 and 0.975 fractile curves) and experimental data

3 INTHERPOL THERMAL FATIGUE TEST [4]

The INTHERPOL testing facility, developed at EDF R&D is designed in order to perform pure thermal fatigue test on tubular specimen under mono-frequential thermal loading. INTHERPOL specimens consist of a pipe comprising a circumferential welding located at a distance of 200 mm from the bottom of the specimen (Figure 2). According to the test considered, the mock-up specimens can have various types of industrial surface finish. Thermal cycling is carried out on a 70 mm broad sector of the internal surface of the specimen. The cold shock is generated by the pulverization of a cold-water spray while the hot shock is obtained by submitting the cooled surface to an intense infrared radiation.

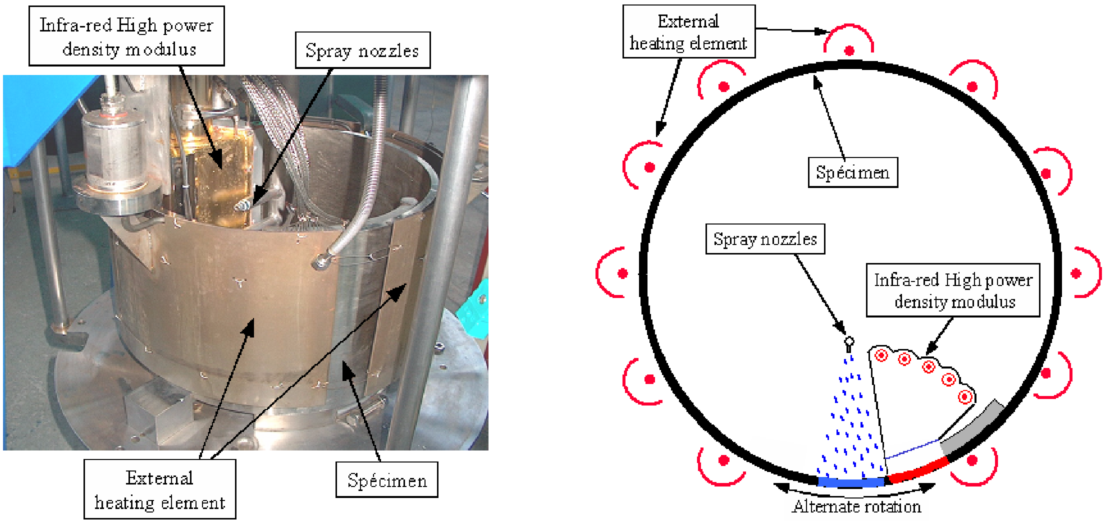

Figure 2. View of the testing facility and principle of operation

Four tests have been carried out with thermal amplitude of 120 to 140°C and a cycle duration ranging of 5 to 8 seconds on mock-up including several surface finishes. Each fatigue test is stopped periodically in order to detect the initiation and the evolution of the cracks by intermediate die penetrated inspections. During the test, the temperature is measured by thermocouples. The recorded temperatures are used as input data to perform finite element simulations in order to obtain the strain and stress fields that are necessary to interpret experimental results.

The structural thermal fatigue database comprises 12 experimental points, including 4 right censored observations. These data are used together with the random specimen fatigue model (section 2) to build a complete structural random fatigue model.

4 IDENTIFICATION OF A SPECIMENS-TO-STRUCTURE PASSAGE FACTOR

4.1 Introduction

Fatigue life tests are usually performed in idealized conditions in laboratory. However, actual structures are subject to varying environment and sometimes hard conditions. In the existing nuclear code [7], the factors $\gamma^N = 20$ on number-of-cycles *N* (for a low cycle fatigue (LCF) regime) and $\gamma^S = 2$ for stress level *S* (for a high cycle fatigue (HCF) domain) are introduced to relate the number-of-cycles to failure of laboratory test specimens to the designed number-of-cycles to failure of actual reactor components. More precisely, the design lifetime of a component reads:

$$N_{design}^{struct}\left(S\right) = \min\left(\frac{N^{spec}\left(S\right)}{20}, N^{spec}\left(2S\right)\right) \tag{14}$$

These passage factors account empirically for the effects of size, the surface finish, the industrial environment of the actual structure and the scatter of experimental data.

The literature shows that these factors are not well known. Moreover, specimens and structures are made of the same material but a realization of a specimen is independent from a given structure since it is not extracted from the structure. For these reasons, the passage factors can be regarded as random variables, which have to be determined by a probabilistic identification using structural fatigue data.

Results of structural thermal fatigue are obtained in the HCF domain. The effect of γ^N can be neglected and then only γ^S has to be identified. In the following section, an original method is proposed to evaluate the random passage factor $\gamma^S\left(\omega\right)$ by considering the specimen fatigue model developed in section 2 and the INTHERPOL fatigue data (section 3).

4.2 Polynomial chaos representation of the passage factor

Let $\left(\Omega, F, P\right)$ be a probability space, where Ω is the sample space, F is the σ-algebra of subsets of Ω and P is a probability measure. Let $X(\omega)$ be a real random variable, whose probability density function (PDF) is denoted by $f_X(x)$. It can be shown that any random variable in this space can be cast as a polynomial series expansion in a standard normal variable ξ:

$$X(\omega) = \sum_{i=0}^{\infty} a_i H_i\left(\xi\left(\omega\right)\right) \tag{15}$$

In this expression, $\{H_i(x), i \in N\}$ are the Hermite polynomials, which form an orthogonal family with respect to the Gaussian probability measure $\varphi(x) = \exp(-x^2/2)/\sqrt{2\pi}$:

$$\left\langle H_i, H_j \right\rangle = \int_{-\infty}^{+\infty} H_i\left(x\right) H_j\left(x\right) \varphi\left(x\right) dx = \delta_{ij}\, i! \quad (\delta_{ij} \text{ Kronecker symbol}) \tag{16}$$

In other words, the coefficients $\{a_i, i \in N\}$ in Eq.(15) are the "coordinates" of X in the Hermite polynomial basis. When the PDF of X is prescribed, the PCE coefficients can be computed by projection or regression. In practice, unimodal random variables, such as the passage factor $\gamma^S\left(\omega\right)$, can be accurately approximated using a truncated series expansion $\gamma^S = \sum_{i=0}^{p} a_i H_i\left(\xi\left(\omega\right)\right)$ where $p = 3$ is usually sufficient [8].

4.3 Non parametric maximum likelihood estimation

Using the polynomial chaos representation of the passage factor $\gamma^S = \sum_{i=0}^{p} a_i H_i\left(\xi\left(\omega\right)\right)$ and for a given vector of PCE coefficients ***a***, one can generate realizations of the logarithm of the structural number-of-cycles to failure $\ln(N^{stru})$, which is defined as the random counterpart of Eq.(14):

$$\begin{aligned} \ln\left(N^{stru}\left(S,\omega\right)\right) &= \ln\left(N^{spec}\left(\gamma^S\left(\omega\right)S,\omega\right)\right) \\ &= \left[A\ln\left(\gamma^S\left(\omega\right)S - E\left(\omega\right)\right) + B\right]\left[1 + \delta u\left(\omega\right)\right] \end{aligned} \tag{17}$$

where A, B, δ and $E\left(\omega\right)$ have been identified with the specimen fatigue model (section 2).

Let $(\ln N_1,\ldots,\ln N_K)$ be an independent and identically distributed simulation sample of $\ln\left(N^{stru}(S,\omega)\right)$, obtained from Eq.(17), by sampling $E(\omega)$, $u(\omega)$ and $\gamma^S(\omega)=\sum_{i=0}^{p} a_i H_i(\xi(\omega))$. The kernel approximation of the PDF [9] of $\ln\left(N^{stru}(S,\omega)\right)$ is given by:

$$\hat{f}_{\ln N^{stru}}(N,S,\boldsymbol{a})=\frac{1}{K}\sum_{k=1}^{K}\frac{1}{h}K\left(\frac{\ln N-\ln N_k^{stru}(S,\boldsymbol{a})}{h}\right) \tag{18}$$

where K is a *kernel function* and h is a positive smoothing parameter, called the *bandwidth*. In practice, K is chosen to be a unimodal PDF that is symmetric about zero (e.g. Gaussian standard PDF, triangular PDF, etc.). The kernel estimate is constructed by combining contributions from each data point.
Here, the standard normal PDF is chosen to be the kernel function. Then, Eq.(18) rewrites:

$$\hat{f}_{\ln N^{stru}}(N,S,\boldsymbol{a})=\frac{1}{K}\sum_{k=1}^{K}\frac{1}{h}\varphi\left(\frac{\ln N-\left[A\ln\left(\sum_{i=0}^{p}a_iH_i(\xi_k)S-E_k\right)+B\right][1+\delta u_k]}{h}\right) \tag{19}$$

and the CDF of $\ln\left(N^{stru}(S,\omega)\right)$ is given by:

$$\hat{F}_{\ln N^{stru}}(N,S,\boldsymbol{a})=\frac{1}{K}\sum_{k=1}^{K}\frac{1}{h}\Phi\left(\frac{\ln N-\left[A\ln\left(\sum_{i=0}^{p}a_iH_i(\xi_k)S-E_k\right)+B\right][1+\delta u_k]}{h}\right) \tag{20}$$

where Φ is the standard normal CDF (see Eq.(11)).

Let $\boldsymbol{N}=\{(N_1,S_1);\ldots;(N_Q,S_Q)\}$ be now an experimental fatigue data set. The likelihood function of the data set $\boldsymbol{N}$ reads:

$$L_Q(\boldsymbol{a};\boldsymbol{N})=\prod_{q=1}^{Q}\left\{\left[\hat{f}_{\ln N^{stru}}(N_q,S_q,\boldsymbol{a})\right]^{\alpha_q}\left[1-\hat{F}_{\ln N^{stru}}(N_q,S_q,\boldsymbol{a})\right]^{1-\alpha_q}\right\} \tag{21}$$

where:

$$\alpha_q=\begin{cases}1 \text{ if } N_q \text{ is a failure}\\ 0 \text{ if } N_q \text{ is a censored observation}\end{cases}$$

In order to estimate the unknown coefficients $\{a_1,\ldots,a_p\}$, one has to solve the following maximum likelihood optimization problem:

$$\boldsymbol{a}^*=\arg\min_{\boldsymbol{a}\in\mathbf{R}^{p+1}}\left(-\ln\left[L_Q(\boldsymbol{a};\boldsymbol{N})\right]\right) \tag{22}$$

Practically speaking, the usual gradient-based algorithm fail to converge due to the fact that the function to optimize present flat regions. The CONDOR (COnstressed, Non-linear, Direct, parallel Optimization using trust Region method for high-computing load function), developed by Vanden Berghen and Bersini [10], appears to be useful in the present case.

4.4 Results

The proposed maximum likelihood method has been applied to the INTHERPOL fatigue database obtained on tubular mock-ups (section3). The two first statistical moments (i.e mean and standard deviation) of the passage factor can be computed from the identified PDF. Estimates obtained from the PCE coefficients are reported in Table 1 together with the estimated PCE coefficients.

A few data points have been used to identify the factor passage and they show little scattering. Moreover, the whole structural fatigue data operate in a concentrated HCF domain, i.e. where low stress amplitudes are applied. As a consequence, the resulting PDF of the passage factor (Figure 3) is very peaked: $\gamma^S(\omega)$ could be considered as deterministic parameter set equal to 1.26.

Table 2. Numerical results for the identification of γ^S

Polynomial chos coefficients			
$a_0 = 1.2604$	$a_1 = 0.0296$	$a_2 = 0.0069$	$a_3 = 0.0354$

Mean $\hat{m} = 1.2604$ Standard deviation $\hat{\sigma} = 0.0466$

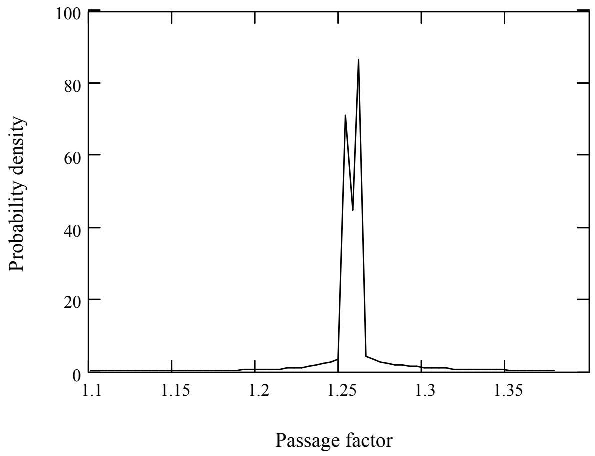

Figure 3. PDF of the identified passage factor γ^S

Figure 4 shows specimen and structure quantile plots of *S-N* curves. By considering a 95 % confidence interval around each median model *S-N* curve, we can see that each probabilistic model give different fractile curves. The updated confidence interval (i.e. for structural data) satisfactorily encompasses the INTHERPOL data. However, both confidence intervals show the same scattering since the passage factor is almost deterministic. The random number-of-cycles to failure seems to be well represented by the proposed model since all data points except 3 are included in the interval.

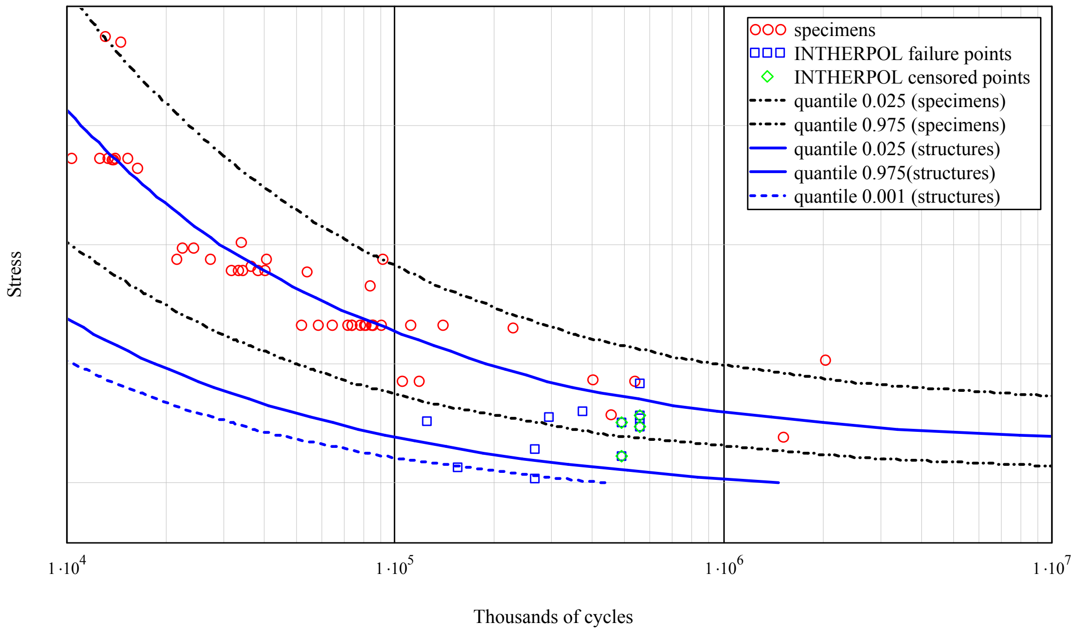

Figure 4. *S-N* plots for the specimen and structural data with 95% confidence intervals

The weak point of the method deals with the existence of the endurance limit E, which is still a subject of debate. In the proposed fatigue model, this parameter is considered as random and its PDF does not allow one to generate realizations close to small stress values, such as those observed for the structural thermal fatigue data. This implies that 2 data points corresponding to low stress amplitudes are not included in the 95% confidence interval. However, these points seem to be realizations of the 0.001 quantile of the *S-N* curve, as seen from Figure 4, and then validate the identified fatigue model.

5 CONCLUSION

The paper proposes a method to derive probabilistic *S-N* curves for structures based on a) a large data base of experimental points obtained from fatigue samples; b) a data base of experimental points obtained by testing mock-up structures (tubular mock-ups with welds).

The first part of the paper extends a methodology proposed formerly to derive probabilistic *S-N* curves from samples. The obtained model considers a lognormal distribution for the sample number-of-cycles-to-failure, whose standard deviation is varying with stress level S (heteroscedasticity assumption), and whose mean curve has an endurance limit. The latter is also considered as a lognormal random variable, whose parameters are identified.

In a second part, experimental results obtained on the INTHERPOL facility are exploited in order to assess a “passage factor”, which is a multiplicative factor γ^{s} to be applied to the stress level S in order to obtain the number-of-cycles-to failure of the mock up structure.

A general probabilistic inverse method is developed that makes use of the polynomial chaos representation of the passage factor random variable. Then the method is applied using 12 experimental points obtained on the INTHERPOL facility. It appears that the passage factor for the mock-up structure under consideration is about 1.26 and has little scatter. Note that these results should be used per se, since the number of INTHERPOL data points is low and the points are grouped in a specific region of the *S-N* domain.

As a post-processing, the PDF of the number-of-cycles-to-failure of the mock-up structure is derived. The associated 95%-confidence interval encompasses satisfactorily the INTHERPOL data points. Further application of the method is required when additional mock-up data points involving other experimental conditions (surface finish, shape of welds) are available.

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